Power Loss Minimization Using Optimal Power Flow Based on Particle Swarm Optimization

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Abstract—This paper describes optimal power flow based on particle swarm optimization in which the power transmission loss function is used as the problem objective. Although most of optimal power flow problems involve the total production cost of the entire power system, in some cases some different objective may be chosen. In this paper, to minimize the overall power losses four types of decision variables are participated. They are i) power generated by power plants, ii) specified voltage magnitude at control substations, iii) tap position of on-load tap-changing transformers and iv) reactive power injection from reactive power compensators. Particle swarm optimization (PSO) is well-known and widely accepted as a potential intelligent search methods for solving such a problem. Therefore, PSO-based optimal power flow is formulated and tested in comparison with quasi-Newton method (BFGS), genetic-based (GA-based) optimal power flow. For test, a 6-bus and 30-bus IEEE power system are employed. As a result, the PSO-based optimal power flow gives the best solutions over the BFGS and the GA-based optimal power flow methods.

I. INTRODUCTION

Optimal power flow is one of nonlinear constrained and occasionally combinatorial optimization problems of power systems. The various algorithms for solving such problems can be found in the literature. The optimal power flow problem has been developed continually since its introduction by Carpentier in 1962 [1]. It is useful to determine the goals of optimal power flow problems. The primary goal of a generic optimal power flow is to minimize the total production costs of the entire system to serve the load demand for a particular power system while maintaining the security of the system operation. The production costs of electrical power systems may depend on the situation, but in general they normally mean to the cost of generating power at each generating unit of power plants. This operation is subjected to keep each device in the power system within its desired operation range at steady-state. This will include maximum and minimum outputs for generators, maximum MVA flows of power transmission lines and transformers, as well as system bus voltages within specified ranges.

It has taken over decades to develop efficient algorithms for its solution because it is a very large, non-linear mathematical programming problem. Many different mathematical approaches have been applied for seeking its solution. The methods discussed in the literature use one of the following five methods [2]. They are i) lambda iteration method as found in economic dispatch problem solving, ii) gradient method, iii) Newton’s method, iv) linear programming and v) interior point method. Apart from analytical approaches, there also exist heuristic search methods.

Heuristic search methods (e.g. simulated annealing [3], genetic algorithm [4], evolutionary programming [5], particle swarm optimization [6]-[9], etc) have been recently released for the optimal power flow problem. The genetic algorithm (GA) based solution methods are found to be most suitable because of their ability of simultaneous multidimensional search for optimal solution. They are well-known and widely used at the beginning period of solving the optimal power flow problems based on heuristic search methods. However, recent literatures show some deficiency of GA-based methods, newly developed heuristic approaches called particle swarm optimization (PSO) has been introduced [10]. This method combines social psychology principles and evolutionary computation to motivate the behavior of organisms such as fish schooling, bird flocking, etc. PSO has been discovered to have better convergence performances than GAs [11]. The PSO-based method is based on a metaphor of social interaction. It searches a space by adjusting the trajectories of individual vectors, called ‘particles’ of a swarm, as they are conceptualized as moving as points in multidimensional space. The individual particles are drawn stochastically towards the positions of their own previous best performances and the best previous performance of the entire swarm.

This paper proposes an application of PSO to solve optimal power flow problems. The controllable system quantities are generator MW, controlled voltage magnitude, reactive power injection from reactive power sources and transformer tapping. The objective use herein is to minimize the power transmission loss by optimizing the control variables within their limits. Therefore, no violation on other quantities (e.g. MVA flow of transmission lines, load bus voltage magnitude, generator MVAR) occurs in normal system operating conditions. The proposed PSO-based method has been tested on a six-bus test system [12] and compared with the GA-based method.

This paper organizes a total of five sections. Next section, Section II illustrates optimal power flow problems with corresponding mathematical expressions of its objective and various practical constraints. Section III gives the brief of two heuristic search methods, GA and PSO for comparative purposes. It also provides the algorithm procedure, described
step-by-step. Section IV is the simulation results and discussion. Conclusion remark is in Section V.

II. OPTIMAL POWER FLOW PROBLEMS

A. Problem Formulation

The optimal power flow problem is a nonlinear optimization problem. It consists of a nonlinear objective function defined with nonlinear constraints. The optimal power flow problem requires the solution of nonlinear equations, describing optimal and/or secure operation of power systems. The general optimal power flow problem can be expressed as a constrained optimization problem as follows.

Minimize \( f(x) \)

Subject to \( g(x) = 0 \), equality constraints
\( h(x) \leq 0 \), inequality constraints

By converting both equality and inequality constraints into penalty terms and therefore added to form the penalty function.

\[
P(x) = f(x) + \Omega(x)
\]

\[
\Omega(x) = \rho \{g^2(x) + [\max(0, h(x))]^2\}
\]

Where
\( P(x) \) is the penalty function
\( \Omega(x) \) is the penalty term
\( \rho \) is the penalty factor

By using a concept of the penalty method [13], the constrained optimization problem is transformed into an unconstrained optimization problem in which the penalty function as described above is minimized.

B. Objective Function

Although most of optimal power flow problems involve the total production cost of the entire power system, in some cases some different objective may be chosen. In this paper, the power transmission loss function is set as the objective function. The power transmission loss can be expressed as follows.

\[
F_{loss} = \sum_{i,j} N_{ij} \left\{ V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j) \right\}
\]

Where
\( V_i \) is the voltage magnitude at bus \( i \)
\( g_{ij} \) is the conductance of line \( i-j \)
\( \delta_i \) is the voltage angle at bus \( i \)
\( N_{ij} \) is the total number of transmission lines

C. System Constraints

The controllable system quantities are generator MW, controlled voltage magnitude, reactive power injection from reactive power sources and transformer tapping. The objective use herein is to minimize the power transmission loss function by optimizing the control variables within their limits. Therefore, no violation on other quantities (e.g. MVA flow of transmission lines, load bus voltage magnitude, generator MVAR) occurs in normal system operating conditions. These are system constraints to be formed as equality and inequality constraints as shown below.

1) Equality constraint: Power flow equations

\[
P_{G_i} - P_{D_i} - \sum_{j=1}^{N_B} V_i \| V_j \| Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) = 0
\]

\[
Q_{G_i} - Q_{D_i} + \sum_{j=1}^{N_B} V_i \| V_j \| Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) = 0
\]

Where
\( P_{G_i} \) is the real power generation at bus \( i \)
\( Q_{G_i} \) is the reactive power generation at bus \( i \)
\( P_{D_i} \) is the real power demand at bus \( i \)
\( Q_{D_i} \) is the reactive power demand at bus \( i \)
\( N_B \) is the total number of buses
\( \theta_{ij} \) is the angle of bus admittance element \( i,j \)
\( Y_{ij} \) is the magnitude of bus admittance element \( i,j \)

2) Inequality constraint: Variable limitations

\[
V_{i,\text{min}} \leq V_i \leq V_{i,\text{max}}
\]

\[
T_{i,\text{min}} \leq T_i \leq T_{i,\text{max}}
\]

\[
Q_{\text{comp,i,\text{min}}} \leq Q_{\text{comp,i}} \leq Q_{\text{comp,i,\text{max}}}
\]

\[
P_{G_i,\text{min}} \leq P_{G_i} \leq P_{G_i,\text{max}}
\]

Where
\( V_{i,\text{min}}, V_{i,\text{max}} \) are upper and lower limits of voltage magnitude at bus \( i \)
\( T_{i,\text{min}}, T_{i,\text{max}} \) are upper and lower limits of tap position of transformer \( i \)
\( Q_{\text{comp,i,\text{min}}, Q_{\text{comp,i,\text{max}}} \) are upper and lower limits of reactive power source \( i \)
\( P_{G_i,\text{min}}, P_{G_i,\text{max}} \) are upper and lower limits of power generated by generator \( i \)

The penalty function can be formulated as follows.

\[
P(x) = F_{loss} + \Omega_P + \Omega_Q + \Omega_T + \Omega_{\text{Q,G}} + \Omega_{\text{Q,P}}
\]

Where

\[
\Omega_P = \rho \sum_{i=1}^{N_B} \left\{ P_{G_i} - P_{D_i} - \sum_{j=1}^{N_B} V_i \| V_j \| Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) \right\}^2
\]

\[
\Omega_Q = \rho \sum_{i=1}^{N_B} \left\{ Q_{G_i} - Q_{D_i} + \sum_{j=1}^{N_B} V_i \| V_j \| Y_{ij} \sin(\theta_{ij} - \delta_i + \delta_j) \right\}^2
\]
\[\Omega_c = \rho \sum_{i=1}^{N_c} \left[ \max(0, Q_{\text{comp},i} - Q_{\text{comp}}^\text{max}) \right]^2 + \rho \sum_{i=1}^{N_c} \left[ \max(0, Q_{\text{comp},i} - Q_{\text{comp}}^\text{min}) \right]^2 \]

(13)

\[\Omega_t = \rho \sum_{i=1}^{N_t} \left[ \max(0, T_i - T_i^\text{max}) \right]^2 + \rho \sum_{i=1}^{N_t} \left[ \max(0, T_i^\text{min} - T_i) \right]^2 \]

(14)

\[\Omega_p = \rho \sum_{i=1}^{N_p} \left[ \max(0, V_i^\text{max} - V_i) \right]^2 + \rho \sum_{i=1}^{N_p} \left[ \max(0, V_i^\text{min} - V_i) \right]^2 \]

(15)

\[\Omega_g = \rho \sum_{i=1}^{N_g} \left[ \max(0, P_{\text{ref},i} - P_{\text{ref}}^\text{max}) \right]^2 + \rho \sum_{i=1}^{N_g} \left[ \max(0, P_{\text{ref},i} - P_{G,i}) \right]^2 \]

(16)

III. INTELLIGENT SEARCH METHODS

A. Genetic Algorithm (GA)

There exist many different approaches to adjust the motor parameters. The GA is a well-known [14] there exist a hundred of works employing the GA technique to identify system parameters in various forms. The GA is a stochastic search technique that leads a set of population in solution space evolved using the principles of genetic evolution and natural selection, called genetic operators e.g. crossover, mutation, etc. With successive updating new generation, a set of updated solutions gradually converges to the real solution. Because the GA is very popular and widely used in most research areas where an intelligent search technique is applied, it can be summarized briefly as shown in the flowchart of Fig. 1 [15].

In this paper, the GA is selected to build up an algorithm to solve optimal power flow problems (all generation from available generating units). To reduce programming complication, the Genetic Algorithm (GADS TOOLBOX in MATLAB [16]) is employed to generate a set of initial random parameters. With the searching process, the parameters are adjusted to give the best result.

B. Particle Swarm Optimization (PSO)

Kennedy and Eberhart developed a particle swarm optimization algorithm based on the behavior of individuals (i.e., particles or agents) of a swarm [17]. Its roots are in zoologist’s modeling of the movement of individuals (i.e., fish, birds, and insects) within a group. It has been noticed that members of the group seem to share information among them to lead to increased efficiency of the group. The particle swarm optimization algorithm searches in parallel using a group of individuals similar to other AI-based heuristic optimization techniques. Each individual corresponds to a candidate solution to the problem. Individuals in a swarm approach to the optimum through its present velocity, previous experience, and the experience of its neighbors. In a physical n-dimensional search space, the position and velocity of individual i are represented as the velocity vectors. Using these information individual i and its updated velocity can be modified under the following equations in the particle swarm optimization algorithm.
\[ x_i^{(k+1)} = x_i^{(k)} + v_i^{(k+1)} \]  
(17)

\[ v_i^{(k+1)} = v_i^{(k)} + \alpha_i \left( x_i^{\text{gbest}} - x_i^{(k)} \right) + \beta_i \left( x_i^{\text{gbest}} - x_i^{(k)} \right) \]  
(18)

Where

\[ x_i^{(k)} \] is the individual \( i \) at iteration \( k \)

\[ v_i^{(k)} \] is the updated velocity of individual \( i \) at iteration \( k \)

\[ \alpha_i, \beta_i \] are uniformly random numbers between \([0,1]\)

\[ x_i^{\text{gbest}} \] is the individual best of individual \( i \)

\[ x_i^{\text{gbest}} \] is the global best of the swarm

The procedure of the particle swarm optimization can be summarized in the flow diagram of Fig. 2.

**IV. SIMULATION RESULTS**

To verify the effectiveness of the proposed particle swarm optimization, a six-bus test power system [12] as shown in Fig. 3 was tested. Information of the test power system is given in TABLE I. The simulations were performed using MATLAB software [16]. The test was carried out by solving the optimal power flow problem of the power loss objective in which variable limits as given in TABLE II are used as system constraints. The test power system was loaded with 50 + j10 MVA, 30 + j18 MVA and 55 + j11 MVA for load at bus 1, 2 and 3, respectively. For comparison purposes, BFGS, genetic algorithm and particle swarm optimization were applied to solve this test case. Each method was challenged by solving a given optimal power flow problem of 30 trials randomly. Minimum, average, maximum and standard deviation of the 30 solutions obtained by each method are evaluated are shown in TABLE III and TABLE IV. Also, convergence characteristics of the PSO-based optimal power flow solution are depicted in Fig.4.

![Six-bus test power system](image1)

![Convergence characteristics obtained by PSO](image2)

The results showed that the PSO-based optimal power flow method gave the best results when compared with those obtained by the GA and the BFGS. For 6-bus test system the average power loss solutions are 7.8587 MW, 6.9705 MW and 6.8425 MW for the BFGS, GA and PSO methods, respectively. However, when considering the minimum power loss found,
the BFGS is the method that can find the least cost function of 6.7361 MW. The standard deviations of the solutions obtained by the BFGS, GA and PSO methods are 1.9060, 0.1521 and 0.0759 respectively. For 30-bus test system the average power loss solutions are 40.20 MW, 15.95 MW and 17.72 MW for the BFGS, GA and PSO methods, respectively. However, when considering the minimum power loss found, the GA is the method that can find the least cost function of 10.90 MW. The standard deviations of the solutions obtained by the BFGS, GA and PSO methods are 20.92, 5.83 and 2.39 respectively. This reveals that the PSO method is the most efficient method among the these three methods of solving the optimal power flow problem with the power loss objective.

V. CONCLUSIONS

Solution methods for solving optimal power flow problems with the power transmission loss objective are described in this paper. Some efficient search methods (quasi-Newton method of BFGS, genetic algorithm and particle swarm optimization) are employed. A six-bus power system was established as a test case for benchmarking. The results showed that a set of optimal solutions with respect to the power transmission loss objective can be efficiently found. As a result, the PSO method proves that it can find a place among some efficient search methods in order to find a near global solution of the optimal power flow problems.

REFERENCES


